Bayesian Probabilistic Approach to Structural Health Monitoring

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Abstract: A Bayesian probabilistic methodology for structural health monitoring is presented. The method uses a sequence of identified modal parameter data sets to compute the probability that continually updated model stiffness parameters are less than a specified fraction of the corresponding initial model stiffness parameters. In this approach, a high likelihood of reduction in model stiffness at a location is taken as a proxy for damage at the corresponding structural location. The concept extends the idea of using as indicators of damage the changes in structural model parameters that are identified from modal parameter data sets when the structure is initially in an undamaged state and then later in a possibly damaged state. The extension is needed, since effects such as variation in the identified modal parameters in the absence of damage, as well as unavoidable model error, lead to uncertainties in the updated model parameters that in practice obscure health assessment. The method is illustrated by simulating on-line monitoring, wherein specified modal parameters are identified on a regular basis and the probability of damage for each substructure is continually updated.

INTRODUCTION

Structural health monitoring, or SHM for short, is the process of establishing some knowledge of the current condition of a structure. The ultimate goal is to determine the existence, location, and degree of damage in a structure if damage occurs. A successful technology for SHM has enormous potential for applications in monitoring of offshore structures and bridges subject to fatigue, corrosion, impacts, and earthquakes, as well as buildings and aerospace structures subject to severe loads or structural deterioration (Natke and Yao 1988; Aktan et al. 1997). A great deal of research in the past thirty years has therefore been aimed at establishing effective local and global methods for health monitoring in civil, mechanical, and aerospace structures. An extensive survey of global methods that use vibration characteristics to perform SHM is presented in Doebling et al. (1996). Despite this substantial body of research, SHM remains a technically challenging problem, and no global technique has been well established (Sanayei et al. 1998).

One typical global approach involves comparing structural models identified using sets of modal data (natural frequencies and modeshapes) from a structure before and after damage has occurred. This model−based SHM approach relies on structural model updating methodologies to solve the inverse problem of determining the parameters of a structural model given some modal data (e.g., Mottershead and Friswell 1993; Beck and Katafygiotis 1998; Capecchi and Vestroni 1999; Sanayei et al. 1999). The critical assumption is that changes in the parameters of the structural model imply changes in the parts of the real structure associated with the model parameters. The focus is usually on local loss of stiffness as a proxy for localized damage.

There are some inherent difficulties in the model−based SHM approach. Structural models cannot be expected to predict perfectly the full behavior of the structure. For instance, the model may not account for effects such as thermally-induced diurnal variations and amplitude dependence of the modal parameters. Further, the available measured information is restricted by limits on the amount of instrumentation and the fact that only a few of the lower modes of a structure can be identified with confidence. Also, modal test data tends to show significant variation from one test to the next. Finally, the modal parameters are insensitive to localized changes in the model stiffness parameters. Model error and measurement noise, when combined with this lack of sensitivity, produce ill-conditioning, which can lead to large variations in the identified model parameters that are not due to true changes in the structure. Thus, there is uncertainty in whether or not changes in identified model parameters reflect damage in the structure.

If the structure under consideration is well−characterized by the analytical model and many controlled measurements can be made with very low noise levels, no significant uncertainty may be present, and ignoring the uncertainty may not lead to problems. In many cases, such as with civil structures, these assumptions do not apply, and it is necessary to address the uncertainty. However, there are relatively few papers in the SHM literature in which this uncertainty is explicitly treated using a probabilistic framework (Beck and Katafygiotis 1992; Beck et al. 1994b; Kim and Stubbs 1995; Fares and Maloof 1997; Sohn and Law 1997; Katafygiotis and Lam 1998).

Most methods focus on looking for damage using one set of data from the undamaged structure and another from the structure in a possibly damaged state. In situations where the structure is only measured during infrequent periodic inspections or following a severe loading event for which structural damage is suspected, such methods are potentially useful. Treating the problem in this fashion, however, ignores the long−range monitoring goal of SHM. This goal is to continually monitor a structure so that gradual deterioration, as well as damage from severe events such as impacts and earthquakes, can be detected. Few, if any, methods explicitly consider this continual accumulation of data, although there are a number of advantages to treating SHM as a continual process. First, the effects of noise in the data can potentially be mitigated by using multiple sequential modal tests. Also, by observing the structure continually, systematic changes may be separated from ''random'' fluctuations. Those systematic variations that are not due to damage, such as diurnal thermal effects, can be included in the model to reduce model error. In this manner, gradual damage, such as that due to fatigue and corrosion, may be detected in its earlier stages before there is a risk of catastrophic failure.

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Note. Special Editor: Roger Ghanem. Discussion open until December 1, 2000. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on February 25, 2000. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 126, No. 7, July, 2000. ©ASCE, ISSN 0733-9399/00/0007-0738–0745/$8.00 + $.50 per page. Paper No. 22249.

HEALTH MONITORING METHODOLOGY

Introduction

This paper presents details of a Bayesian probabilistic technique for continual on–line SHM that addresses the ill–conditioning inherent in the inverse problem of detecting stiffness changes from vibration data. Modal data (natural frequencies and incomplete modeshapes) identified from a structure, such as those from ambient or forced vibration tests, are used to identify the model substructure ''stiffness'' parameters. In a deterministic SHM scheme, differences in the stiffness parameters estimated from different modal data sets would be used as indicators of damage. Rather than consider only point estimates for each modal data set, however, the probabilistic method takes uncertainties in the identified model into account by treating the problem within a framework of plausible inference in the presence of incomplete information (Cox 1961; Jaynes 1978). Bayes's theorem is invoked to develop a probability density function (PDF) for the model stiffness parameters conditional on measured modal data and the chosen class of models. Using conditional PDFs derived from sets of modal data determined at different times, a probabilistic damage measure is developed, which addresses the question: Based on the available modal data and acknowledging the unavoidable uncertainties, what is the probability that the current model stiffness parameters are less than a specified fraction of the corresponding undamaged stiffness parameters?

In the following sections, the methodology is presented. First, the modal data and structural model class are defined. Then, the probabilistic framework and the assumptions used therein are described. Within this framework, a damage measure is defined that answers the aforementioned question. Finally, a simple graphical representation of the damage measure and the interpretation of this measure for use in SHM is presented.

Modal Data

A set of N_m modal frequencies, $\hat{\omega}_r$, and N_m generally incomplete modeshapes, $\hat{\psi}_r \in \mathbb{R}^{N_o}$, which are identified from the structure under consideration, are termed the modal data. Here, N_o = number of observed degrees of freedom. These modal data can be identified from ambient or forced vibration data using any reliable modal parameter identification method (e.g., Werner et al. 1987; Beck et al. 1994a; Beck 1996; Katafygiotis and Yuen 2000; Yuen and Katafygiotis 2000). The modal data for the nth test is referred to by $\hat{\Upsilon}_n$. A grouping of modal parameter data sets from N_s different tests is denoted $\mathcal{D} = \{\hat{\Upsilon}_1, \ldots, \hat{\Upsilon}_{N_s}\}$. The data from a structure in a known undamaged state is undamaged data, $\mathcal{D}_{ud}$, while data from the same structure in an unknown state is called possibly damaged data, $\mathcal{D}_{pd}$.

Structural Model Class

The structural model class, $\mathcal{M}$, is based on N_d degree-of-freedom (DOF) linear structural models parameterized by the model parameters $\theta \in \mathbb{R}^{N_\theta}$. The modal frequencies ω_r and modeshapes ϕ_r, $r = 1, \ldots, N_d$, are governed by the eigenvalue equation, $K(\theta)\phi_r = \omega_r^2 M\phi_r$, where M and K = mass and stiffness matrix, respectively. It is assumed that the modal tests are performed using low-amplitude vibrations so that the structure does not exhibit highly nonlinear behavior. Therefore, a linear model of the dynamics should be adequate for identification purposes.

A convenient parameterization for the stiffness matrix is

$$K(\theta) = K_0 + \sum_{i}^{N_\theta} \theta_i K_i \tag{1}$$

where each K_i is a nominal substructure contribution to the global stiffness matrix which could come from a finite-element model of the undamaged structure. The nominal stiffness matrix would then be $K(\theta_0)$ where $\theta_0 = [1, \ldots, 1]$. The mass matrix could be parameterized using a similar substructuring approach, but the writers assume here that it is known with sufficient accuracy from structural drawings so that θ denotes only stiffness-related parameters.

Bayesian Probabilistic Framework

The probabilistic SHM framework presented in this paper utilizes work done by Beck and coworkers (e.g., Beck 1989; Beck and Katafygiotis 1998; Katafygiotis and Beck 1998; Katafygiotis et al. 1998) in probabilistic system identification based on a system of plausible inference for incomplete information (Cox 1961; Jaynes 1978). In this framework, conditional probabilities are interpreted as measures of the plausibility of certain statements given other statements. Bayes' theorem is used in this context to express the updated probabilities of the model parameters θ given some measured data:

$$p(\theta|\mathcal{D}, \mathcal{M}) = cp(\mathcal{D}|\theta, \mathcal{M})p(\theta|\mathcal{M}) \tag{2}$$

where $p(\theta|\mathcal{D}, \mathcal{M})$ is the PDF of the model parameters given the modal data $\mathcal{D}$ and the model assumptions $\mathcal{M}$; c is a normalizing constant; $p(\theta|\mathcal{M})$ is the initial (''prior'') PDF of the model parameters based on engineering and modeling judgment; and $p(\mathcal{D}|\theta, \mathcal{M})$ is the PDF of the modal data given the model parameters. Here, the modeling assumptions $\mathcal{M}$ include those that are used to infer or derive the probability distributions $p(\theta|\mathcal{M})$ and $p(\mathcal{D}|\theta, \mathcal{M})$, as well as the structural modeling assumptions. The theory that follows shows how the distribution $p(\theta|\mathcal{D}, \mathcal{M})$ in (2) is developed and applied in order to determine the likelihood of a local reduction in stiffness.

Modal Parameter PDF

We first develop a model for the PDF $p(\mathcal{D}|\theta)$ in (2), where the dependence on $\mathcal{M}$ is dropped in order to simplify the notation. Using the axioms of probability, and taking $\mathcal{D}_{N_s} = \{\hat{\Upsilon}_1, \ldots, \hat{\Upsilon}_{N_s}\}$ as previously defined

$$p(\mathcal{D}_{N_s}|\theta) = p(\hat{\Upsilon}_{N_s}|\mathcal{D}_{N_s-1}, \theta)p(\mathcal{D}_{N_s-1}|\theta) = \cdots = \prod_{n=1}^{N_s} p(\hat{\Upsilon}_n|\mathcal{D}_{n-1}, \theta) \tag{3}$$

where $p(\hat{\Upsilon}_1|\mathcal{D}_0, \theta) = p(\hat{\Upsilon}_1|\theta)$. Furthermore, it is assumed that $p(\hat{\Upsilon}_n|\mathcal{D}_{n-1}, \theta) = p(\hat{\Upsilon}_n|\theta)$; that is, the user's uncertainty in the nth modal data when a structural model is specified by θ is not influenced by the previous modal data. Thus, (2) becomes

$$p(\theta|\mathcal{D}_{N_s}) = cp(\theta) \prod_{n=1}^{N_s} p(\hat{\Upsilon}_n|\theta) \tag{4}$$

This result shows that within the Bayesian framework, new data can be incorporated into the PDF for the model parameters in a systematic and consistent fashion by simply extending the product by one term.

The PDF $p(\hat{\Upsilon}_n|\theta)$ in (4) is the distribution for a single modal data set given the model parameters. Given the structural model parameters θ, the modal parameters are taken to be independently distributed from mode to mode and from frequency to modeshape, and therefore

$$p(\hat{\Upsilon}_n|\theta) = \prod_{r=1}^{N_m} p(\hat{\omega}_r|\theta)p(\hat{\psi}_r|\theta) \tag{5}$$

where $p(\hat{\omega}_r|\theta)$ and $p(\hat{\psi}_r|\theta)$ are the distributions for the rth modal frequency and modeshape given θ, respectively.

Modeshape PDF: For the observed modeshapes, the model equation is assumed to be

$$\hat{\psi}_r = a_r \Gamma \phi_r(\theta) + e_{\hat{\psi}_r} \tag{6}$$

where a_r is a scaling factor, the matrix $\Gamma \in \mathbb{R}^{N_o \times N_d}$ picks the observed degrees of freedom from the model modeshape ϕ_r, and $e_{\hat{\psi}_r}$ is the modeshape error. The PDF model for $e_{\hat{\psi}_r}$ is a zero-mean Gaussian PDF with covariance matrix C_r equal to a diagonal matrix with all diagonal elements equal to $\delta_r^2 \|\hat{\psi}_r\|^2$. The Principle of Maximum Entropy (Jaynes 1978) is used as a justification for the choice of a Gaussian distribution for the error distribution. The parameter δ_r reflects the uncertainties in the measured modeshapes. It is computed based on the undamaged data sets as

$$\delta_r^2 = \frac{1}{N_s} \sum_{i=1}^{N_s} \frac{\|\hat{\psi}_r^{(i)} - \bar{\psi}_r\|^2}{\|\hat{\psi}_r^{(i)}\|^2} \tag{7}$$

where $\hat{\psi}_r^{(i)}$ is the rth modeshape of the ith set of measurement and $\bar{\psi}_r$ is the averaged modeshape for the rth mode. The resulting PDF for $\hat{\psi}_r$ is

$$p(\hat{\psi}_r|\theta) = c_1 \exp\left[-\frac{1}{2}(\hat{\psi}_r - a_r\Gamma\phi_r)^T C_r^{-1}(\hat{\psi}_r - a_r\Gamma\phi_r)\right] \tag{8}$$

A rational choice for the scaling factor a_r is to take the value that makes $\hat{\psi}_r$ most likely for a given θ; that is

$$a_r = \frac{\langle \hat{\psi}_r, \Gamma\phi_r \rangle}{\|\Gamma\phi_r\|^2} \tag{9}$$

Here, $\langle \cdot, \cdot \rangle$ is the Euclidean inner product between the two vectors, and $\|\cdot\|$ is the Euclidean norm. Substituting (9) into (8) leads to

$$p(\hat{\psi}_r|\theta) = c_1 \exp\left[-\frac{1}{2}\frac{1}{\delta_r^2}\left\|\frac{\hat{\psi}_r}{\|\hat{\psi}_r\|} - \frac{\langle \hat{\psi}_r, \Gamma\phi_r \rangle}{\|\hat{\psi}_r\|\,\|\Gamma\phi_r\|}\frac{\Gamma\phi_r}{\|\Gamma\phi_r\|}\right\|^2\right] \tag{10}$$

Assuming that the partial modeshape $\hat{\psi}_r$ has been normalized to have unit norm, the modeshape PDF can also be expressed as

$$p(\hat{\psi}_r|\theta) = c_1 \exp\left[-\frac{\phi_r^T\Gamma^T(I - \hat{\psi}_r\hat{\psi}_r^T)\Gamma\phi_r}{2\delta_r^2\|\Gamma\phi_r\|^2}\right] \tag{11}$$

Frequency PDF: For the observed frequencies, the model equation is

$$\hat{\omega}_r^2 = \omega_r^2(\theta) + e_{\hat{\omega}_r^2} \tag{12}$$

where the PDF model for the frequency error $e_{\hat{\omega}_r^2}$ is a zero-mean Gaussian PDF with variance ε_r^2. The parameter ε_r is the standard deviation of the squared circular frequencies identified from the undamaged data. The modal error is formed based on the square of the frequency rather than the frequency itself, because the former is the eigenvalue corresponding to the eigenvector ϕ_r used in (6). The resulting PDF for $\hat{\omega}_r^2$ is

$$p(\hat{\omega}_r^2|\theta) = c_2 \exp\left[-\frac{1}{2}\left(\frac{\hat{\omega}_r^2 - \omega_r^2(\theta)}{\varepsilon_r}\right)^2\right] \tag{13}$$

Initial PDF

The initial PDF on the model parameters, θ, is assumed to have the form

$$p(\theta) = c_3 \exp\left[-\frac{1}{2}(\theta - \theta_0)^T \mathbf{S}^{-1}(\theta - \theta_0)\right] \tag{14}$$

which is a joint Gaussian distribution with mean $\theta_0 \in \mathbb{R}^{N_\theta}$ and covariance matrix $\mathbf{S} \in \mathbb{R}^{N_\theta \times N_\theta}$. The choice for θ_0 will generally be $[1, \ldots, 1]^T$ to reflect that the nominal structural model is the most probable model in the absence of any data. The individual parameters will be assumed to be independent, making $\mathbf{S}$ a diagonal matrix of variances, σ_i^2. The choice for σ_i reflects the level of uncertainty in the nominal model.

Final Form of Model Parameter PDF

Using (5), (11), (13), and (14) in (4) leads to the final form of $p(\theta|\mathcal{D})$:

$$p(\theta|\mathcal{D}) = c \exp\left[-\frac{1}{2}J(\theta)\right] \tag{15}$$

where the overall measure of fit (MOF), $J(\theta)$, is

$$J(\theta) = (\theta - \theta_0)^T S^{-1}(\theta - \theta_0) + \sum_{r=1}^{N_m} J_r(\theta) \tag{16}$$

the modal measure of fit (MMOF), $J_r(\theta)$, is

$$J_r(\theta) = \sum_{n=1}^{N_s}\left[\frac{(\hat{\omega}_r^2(n) - \omega_r^2(\theta))^2}{\varepsilon_r^2} + \frac{\phi_r^T\Gamma^T(I - \hat{\psi}_r(n)\hat{\psi}_r^T(n))\Gamma\phi_r}{\delta_r^2\|\Gamma\phi_r\|^2}\right] \tag{17}$$

and c is a normalizing constant.

Although the PDF in (15) gives a nonzero probability for nonphysical negative stiffness values, the amount of probability volume less than zero is generally negligible, so truncation of the PDF for negative values followed by a renormalization is not necessary.

Marginal Distributions

The PDF for θ in (15) is integrated to get the marginal distribution for each structural model parameter. For each parameter θ_i, $i \in 1, \ldots, N_\theta$, this gives

$$p(\theta_i|\mathcal{D}) = \int p(\theta_i, \theta_{-i}|\mathcal{D})\, d\theta_{-i} \tag{18}$$

where $\theta_{-i} = [\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_{N_\theta}]^T$ and integration is performed over the whole domain of the variables. The integral usually cannot be evaluated analytically. In the globally identifiable case (Beck and Katafygiotis 1998) where the integrand is peaked at a single point $\hat{\theta}$, it is approximated using Laplace's method for asymptotic expansion (Papadimitriou et al. 1997):

$$p(\theta_i|\mathcal{D}) \approx \varphi\left(\frac{\theta_i - \hat{\theta}_i}{\hat{\sigma}_i}\right) \tag{19}$$

where φ = standard Gaussian PDF; $\hat{\theta}$ = most probable model obtained by minimizing $J(\theta)$ in (16); and $\hat{\sigma}_i^2$ = ith diagonal element of $L(\hat{\theta})^{-1}$, where $L(\hat{\theta})$ is the Hessian matrix of $J(\theta)$ evaluated at $\hat{\theta}$. In the locally identifiable case, where the integrand is peaked at multiple discrete points, the expression given in (19) can be generalized as a sum of the contributions from all the "locally optimal" points (Beck and Katafygiotis 1998). One more possibility is the unidentifiable case where there is a manifold of lower dimension than the space of θ_{-i} which contains a continuum of locally optimal points and around which the PDF in (18) is concentrated. This case is difficult to handle in general, and requires special techniques (e.g., Katafygiotis et al. 1998). It is found that in the examples considered here, the integrands are all globally identifiable, and so the other cases are not needed. If the structural model is divided [as in (1)] into too many substructures relative to the amount of modal data, however, global identifiability may be lost. Such cases are left for future work.

Probabilistic Damage Measure

The probability that the stiffness parameter for the *i*th substructure in a possibly damaged state has been reduced by more than 100 d_i% from the undamaged state is

$$P_i^{\text{dam}}(d_i) = P(\theta_i^{pd} < (1 - d_i)\theta_i^{ud} | \mathcal{D}_{ud}, \mathcal{D}_{pd}) \tag{20}$$

where $d_i \in [0, 1]$ is the damage threshold for the *i*th substructure; and P_i^{dam} is called the (probabilistic) damage measure. Here, a subscript or superscript *ud* implies that the quantity corresponds to the undamaged structure. Similarly, a subscript or superscript *pd* refers to the possibly damaged structure. The undamaged data set is taken to be fixed after an initialization phase has been completed, while the possibly damaged set is increased as more data is acquired during the monitoring phase.

Given the data $\mathcal{D}_{ud}$ and $\mathcal{D}_{pd}$, θ_i^{ud} and θ_i^{pd} are assumed to be independently distributed with marginal PDFs $p(\theta_i^{ud}|\mathcal{D}_{ud})$ and $p(\theta_i^{pd}|\mathcal{D}_{pd})$, respectively, defined by (18). This assumption simply reflects that, with information of $\mathcal{D}_{ud}$ and $\mathcal{D}_{pd}$, knowing the undamaged parameters gives no information about the value of the possibly–damaged parameters. Using the Gaussian approximations for the marginal distributions given in (19), P_i^{dam} is therefore approximated by

$$P_i^{\text{dam}}(d_i) \approx \Phi\left(\frac{(1 - d_i)\hat{\theta}_i^{ud} - \hat{\theta}_i^{pd}}{\sqrt{(1 - d_i)^2(\hat{\sigma}_i^{ud})^2 + (\hat{\sigma}_i^{pd})^2}}\right) \tag{21}$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function.

The damage measure is calculated and monitored for each substructure in order to perform SHM. Changes in P_i^{dam}, rather than changes in the $\hat{\theta}_i$, are studied to detect structural damage.

Using P_i^{dam}

In a continual online monitoring process, many modal data sets will be available. As noted, these modal data are grouped into $\mathcal{D}_{ud}$ and $\mathcal{D}_{pd}$. A question arises as to which data from $\mathcal{D}_{pd}$ should be used to form the possibly damaged marginal PDFs to calculate P_i^{dam}. If only the most recently identified modal parameter set is used, then damage could be detected as soon as it occurs. However, the damage measure would be sensitive to noise in these modal parameters and could therefore give rise to misleading conclusions. If many sets of data are used, the effects of noise will be mitigated and so low levels of damage may be detectable. Unfortunately, if damage occurs, the marginal PDF, and thus P_i^{dam}, will be strongly biased by the undamaged data already in $\mathcal{D}_{pd}$, so many additional modal tests will be needed before the damaged data can overcome the bias to indicate the existence of damage. In between these extremes are choices that trade off between sensitivity, noise mitigation, and bias.

Rather than consider only one such choice, P_i^{dam} is calculated for a range of modal parameter sets from $\mathcal{D}_{pd}$. Thus, P_i^{dam} is calculated for the most recent modal data set, the two most recent data sets, and so forth until a limiting number of previous data sets, N_{win}. Each time a new modal data set becomes available, P_i^{dam} is recalculated for all of the different subgroupings of modal parameter sets. The number of modal tests that have been performed is the current monitoring cycle, t_{mon}. The window, k, indicates how many previous modal data sets, starting at t_{mon}, are used to calculate P_i^{dam}. Thus, P_i^{dam} is a function of t_{mon} and k as well as the damage threshold d_i. Calculating $P_i^{\text{dam}}(t_{\text{mon}}, k, d_i)$ for specified values of d_i as both t_{mon} and k vary leads to the novel concept of monitoring the fluctuation of the damage measure as a function of time and the amount of recent modal data used. In testing with simulated data, this approach has shown promise as a means of detecting, locating, and assessing the severity of damage.

Alarm Function P_i^{alarm}

P_i^{dam}, calculated from (21) using k possibly damaged modal data sets, will vary even when no damage is present, because the $\hat{\theta}_i^{pd}$ and $\hat{\sigma}_i^{pd}$ that are identified during monitoring fluctuate as a result of variation in the modal data from one test to the next. It is therefore useful to establish a threshold level for P_i^{dam} for each k that, if exceeded, indicates an abnormally high value of the damage measure has occurred under the hypothesis that the structure is not damaged.

Recall that $\hat{\sigma}_i^{pd}$ is a measure of the uncertainty in the value of θ_i^{pd} for the given data, while $\hat{\theta}_i^{pd}$ is the most probable value of θ_i^{pd} for these data. Simulations show that the identified $\hat{\theta}_i^{pd}$ for different modal data sets is much more sensitive to noise in the modal data than $\hat{\sigma}_i^{pd}$. This is due to the fact that $\hat{\sigma}_i^{pd}$, calculated based on the Hessian matrix of the measure-of-fit function $J(\theta)$ in (16), is more dependent on the sensitivity coefficients (derivatives) of the modal parameters with respect to θ, which vary slowly with θ, than to the noise in the data. When examining the variation of P_i^{dam} in the absence of damage, therefore, it is sufficient to consider only the variation of $\hat{\theta}_i^{pd}$ and to take $\hat{\sigma}_i^{pd}$ as constant and equal to $\hat{\sigma}_i^{ud}\sqrt{N_s^{ud}/k}$. The scaling factor on $\hat{\sigma}_i^{ud}$ is to account for the fact that k undamaged model data sets are used rather than the full N_s^{ud} used to determine $\hat{\sigma}_i^{ud}$.

During monitoring, if the structure is in its undamaged state, the identified $\hat{\theta}_i^{pd}$ should with a large probability lie within the cluster of those $\hat{\theta}_i^{ud}$ identified for the undamaged structure. Suppose that the standard deviation of the variation in $\hat{\theta}_i^{ud}$ identified from a series of k undamaged modal data sets is $\sigma_i^{mod}(k)$. Assuming the structure is undamaged, a "rare" event may then be defined as one when $\hat{\theta}_i^{pd}$ falls below $\hat{\theta}_i^{ud} - \gamma\sigma_i^{\text{mod}}(k)$, where, for instance, $\gamma = 3$. This suggests that an alarm function, $P_i^{\text{alarm}}(k, d_i)$, may be defined as the probability of damage according to (21) when $\hat{\theta}_i^{pd} = \hat{\theta}_i^{ud} - \gamma\sigma_i^{\text{mod}}(k)$ and $\hat{\sigma}_i^{pd} = \hat{\sigma}_i^{ud}\sqrt{N_s^{ud}/k}$, that is

$$P_i^{\text{alarm}}(k, d_i) = \Phi\left(\frac{\gamma\sigma_i^{\text{mod}}(k) - d_i\hat{\theta}_i^{ud}}{\hat{\sigma}_i^{ud}\sqrt{(1 - d_i)^2 + N_s^{ud}/k}}\right) \tag{22}$$

Note that P_i^{alarm} is computed based on the undamaged data sets $\mathcal{D}_{ud}$ and so it is a fixed reference quantity during monitoring based on data sets $\mathcal{D}_{pd}$. With a high probability of approximately $\Phi(\gamma)$ (equal to 99.9% when $\gamma = 3$), the identified value of $\hat{\theta}_i^{pd}$ during monitoring using $\mathcal{D}_{pd}$ will be greater than $\hat{\theta}_i^{ud} - \gamma\sigma_i^{\text{mod}}(k)$ when the structure is undamaged, and so P_i^{dam} will be less than P_i^{alarm}. Therefore, in the absence of damage, P_i^{dam} will exceed P_i^{alarm} only in the "rare" event with probability of approximately $[1 - \Phi(\gamma)]$.

The alarm function can thus be used to test the hypothesis that the structure remains in its original undamaged state during monitoring using data sets $\mathcal{D}_{pd}$. For example, consider the scenario when $P_i^{\text{dam}}(t_{\text{mon}}, k, d_i)$ exceeds $P_i^{\text{alarm}}(k, d_i)$ for some values of k. Since the event of $P_i^{\text{alarm}}(k, d_i)$ being exceeded is very rare if the structure is in its undamaged state, one has to give up the latter hypothesis. Thus, for a given monitoring cycle, if $P_i^{\text{dam}}(t_{\text{mon}}, k, d_i)$ exceeds $P_i^{\text{alarm}}(k, d_i)$ for some k, an alarm is set that the *i*th substructure may be damaged. Contingency measures may be taken when the alarm is set. For example, the frequency of the modal tests may be increased so that more data is provided more rapidly for health assessment.

Of course, the rare event of P_i^{dam} exceeding P_i^{alarm} can occur even when the structure is undamaged because of unusual noise in the data. In this case, a false alarm occurs, and so it is important that additional measures be taken to investigate whether the structure is actually damaged whenever the alarm is set. Notice that smaller values of γ increase the chance of false alarms while larger values increase the possibility of missed alarms. It is observed using simulated data that

$P_i^{dam}(t_{mon}, k, d_i)$ has characteristic behaviors depending on whether the structure is undamaged or damaged. These patterns of behavior may be studied as a function of t_{mon} and k in order to establish additional criteria regarding the state of damage that may be applied by a human operator or an expert system when an alarm is set.

Summary of SHM Approach

The proposed method can be applied as an on–line automated structural health monitoring system using the following procedure:

- During an initialization phase, perform many modal tests on the undamaged structure to establish a stable fixed reference PDF for each substructure using (19). Also, use these data to compute the alarm function $P_i^{alarm}(k, \hat{d}_i)$ for each substructure i, window k, and a reference damage threshold $\hat{d}_i$ according to (22).
- Start the monitoring phase, wherein the structure is tested periodically and $P_i^{dam}(t_{mon}, k, \hat{d}_i)$ is calculated from (21) after each new modal test for each substructure i, window k, and the reference damage threshold $\hat{d}_i$.
- If $P_i^{dam}(t_{mon}, k, \hat{d}_i) < P_i^{alarm}(k, \hat{d}_i)$ for all i, k, wait for the next modal test.
- If $P_i^{dam}(t_{mon}, k, \hat{d}_i) \geq P_i^{alarm}(k, \hat{d}_i)$ for some i, k, set an alarm for the ith substructure and increase the frequency of modal tests. Use $P_i^{dam}(t_{mon}, k, d_i)$ for various damage thresholds d_i to assist in determining the likely severity of the damage in substructure i. Using such information and other criteria established through previous investigations, an expert (human or computer system) can be consulted to decide further actions to be taken.

ILLUSTRATIVE EXAMPLES

Consider a 10 DOF shear structure model with story masses m_1, m_2, m_3 = 3 × 10^4 kg; m_4, . . . , m_9 = 2 × 10^4 kg; m_{10} = 10^4 kg; and interstory stiffnesses k_1, k_2, k_3 = 48,000 kN/m; k_4, . . . , k_8 = 44,000 kN/m; and k_9, k_{10} = 40,000 kN/m. The modal data consists of only the first two modes with full modeshapes in each mode. Noisy ''measured'' modal parameters with a 2% coefficient of variation are generated by adding random values chosen from zero-mean Gaussian distributions to the exact modeshapes and exact modal frequencies of 1.17 and 3.12 Hz. The proposed monitoring procedure is implemented using a sequence of these synthetically generated modal data. The reference damage threshold $\hat{d}_i$ is set at 10% for all substructures. The value for γ is taken to be 3. The actual structure sustains damage of 20% stiffness loss in the fifth story, between the 60th and 61st monitoring cycle.

Figs. 1 and 2 correspond to monitoring of the fifth (damaged) and fourth (undamaged) story stiffness, respectively. These figures show $P_i^{dam}(t_{mon}, k, \hat{d}_i)$ and $P_i^{alarm}(k, \hat{d}_i)$ as the monitoring cycle and window vary. Recall, an increase in the monitoring cycle, t_{mon}, indicates that another modal parameter set has been measured, and an increase in the window parameter, k, indicates that more of the previous possibly damaged data is being used to calculate P_i^{dam}. The first frame in Fig. 1, monitoring cycle 61, shows that the alarm threshold is exceeded for many values of k. In an operational scenario, the alarm would be set at this monitoring cycle that damage in the fifth story was likely, the data collection rate would be increased, and the damage measure, P_5^{dam} would be calculated for a range of d_i to evaluate the degree of damage. In contrast, P_4^{dam} in Fig. 2 does not consistently exceed its alarm threshold, so there is no reason to suspect damage there.

Figs. 3 and 4 show the damage measure for the fifth and fourth stories for damage thresholds d_i = 5%, 10%, 15%, and

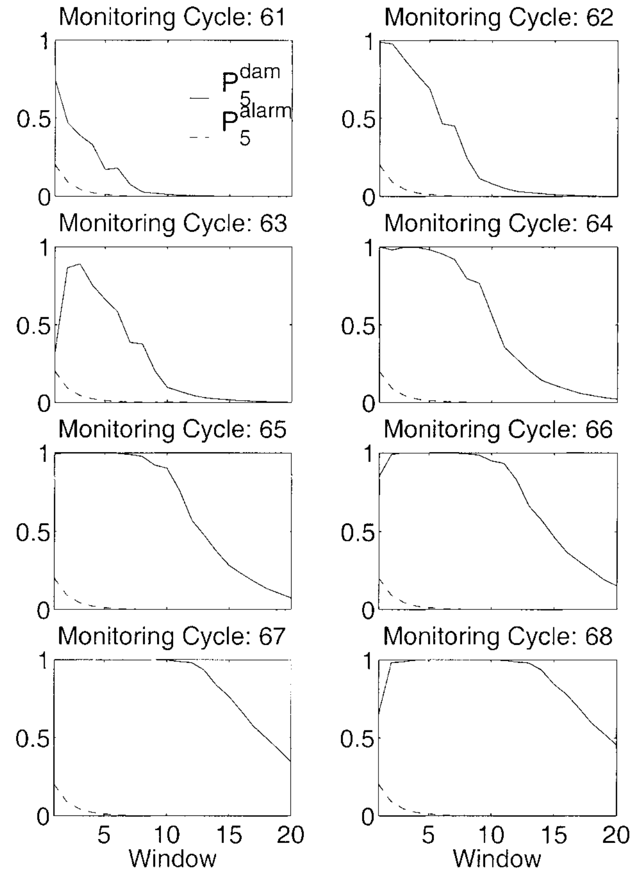

FIG. 1. Monitoring History for P_5^{dam} with Damage Threshold $\hat{d}_5$ = 10%

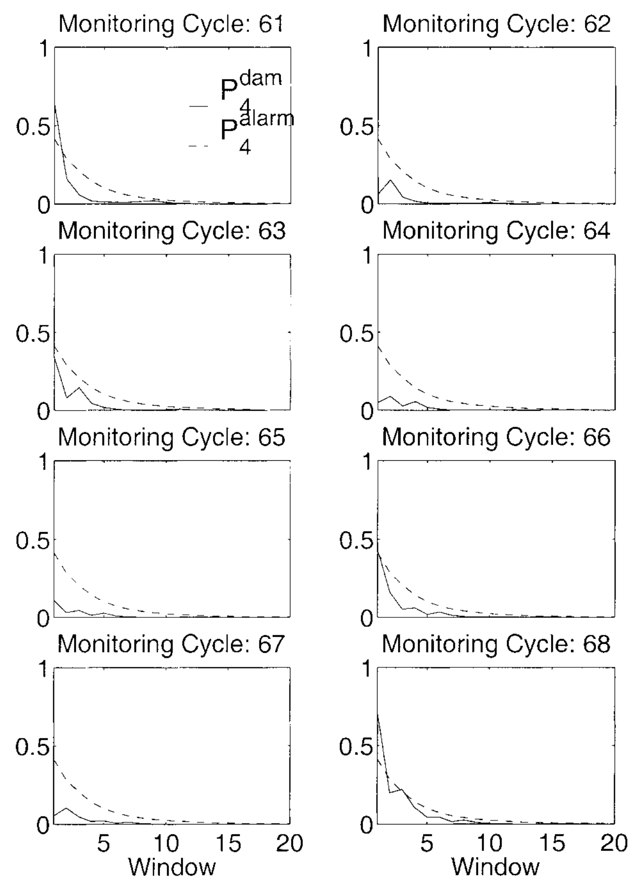

FIG. 2. Monitoring History for P_4^{dam} with Damage Threshold $\hat{d}_4$ = 10%

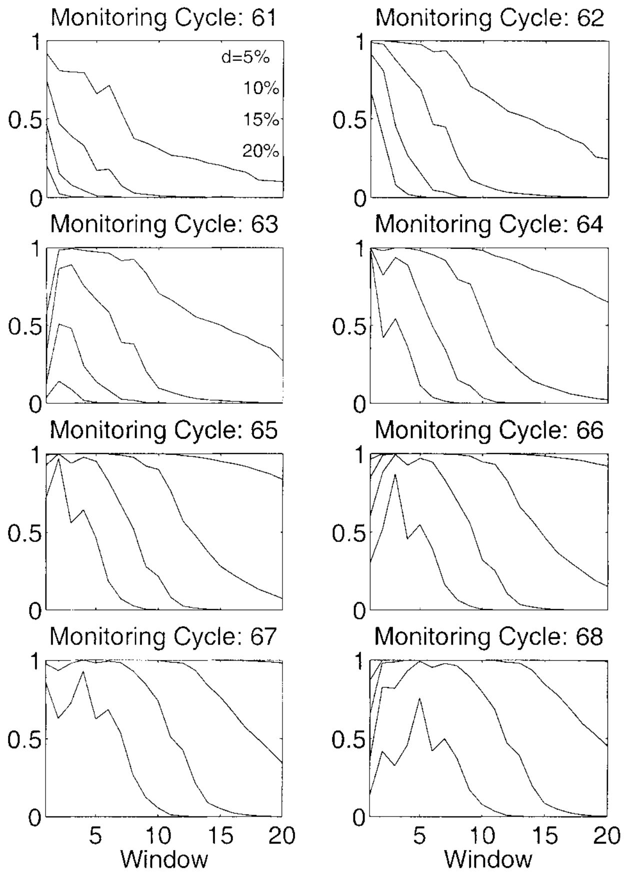

FIG. 3. Monitoring History for P_5^{dam} for Damage Thresholds d_5 = 5, 10, 15, 20%

20%. To avoid biasing the conclusions with data that are believed to correspond to an undamaged state, evaluation of the degree of damage should only consider P_5^{dam} based on data acquired after the alarm was set. For example, at monitoring cycle 65, there are four additional sets of data taken after the alarm was set (at monitoring cycle 61). Thus, only the values of P_i^{dam} for window $k \leq 5$ should be considered for degree-of-damage assessment, since the values for $k > 5$ are calculated with the data before the alarm was set, which would tend to bias the damage measure towards the no-damage side. This is evident from the plots in Fig. 3, where the damage measure generally decreases for windows k larger than the difference between the current monitoring cycle and 61 (at which the alarm was set). The damage measures for d_5 = 5%, 10%, and 15% in Fig. 3 are quite high for different monitoring cycles, while the damage measure for d_5 = 20% is not as large. These indicate that there is a high probability of a damage level exceeding 15% stiffness loss in the fifth story, but not exceeding 20% loss. On the other hand, the damage measures in Fig. 4 are generally small for different monitoring cycles and damage thresholds, indicating there is no substantial damage (<5%) in the fourth story.

Figs. 5 and 6 show the variation of damage measure for a larger number of monitoring cycles. The results in Fig. 5 support the conclusion drawn from the first eight monitoring cycles (Fig. 3) that there is high probability of damage in the fifth story of stiffness loss between 15 and 20%. Note that for window widths greater than 5, say, the damage measures for d_5 = 5%, 10%, and 15% are consistently high for different monitoring cycles. The damage measures for window widths less than 5 tend to fluctuate for different monitoring cycles. The stability of the damage measure for larger window widths as compared with smaller ones demonstrates the benefit of

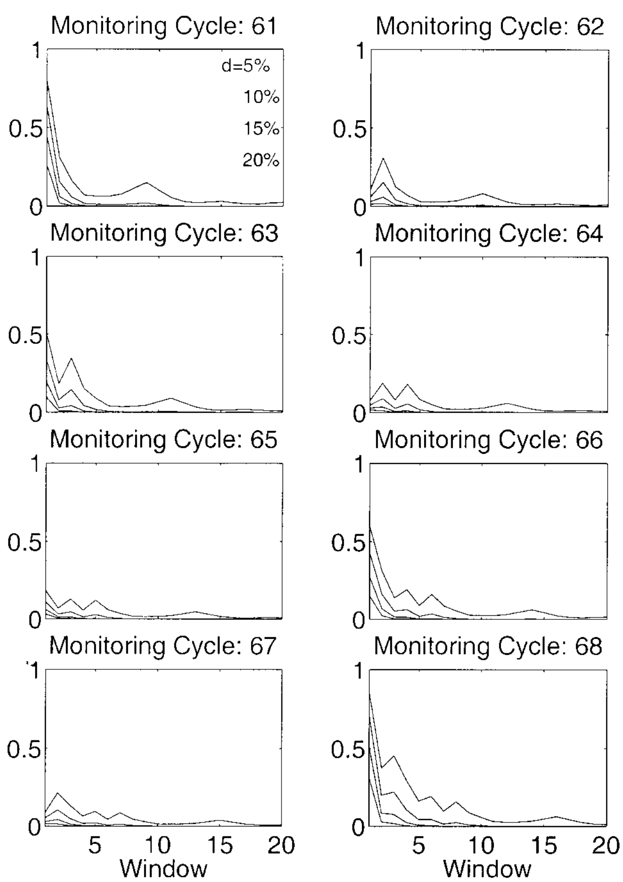

FIG. 4. Monitoring History for P_4^{dam} for Damage Thresholds d_4 = 5, 10, 15, 20%

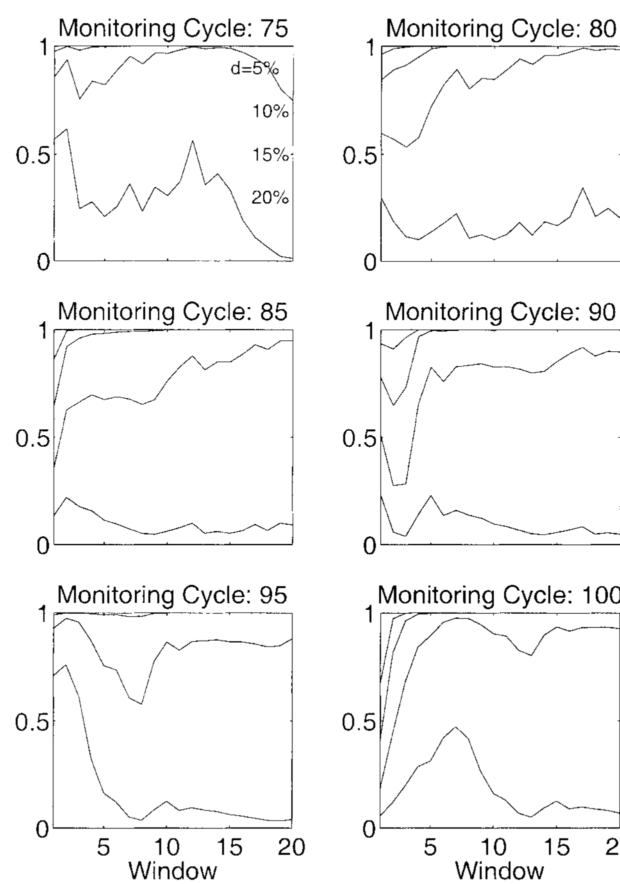

FIG. 5. Monitoring History for P_5^{dam} for Larger Number of Monitoring Cycles

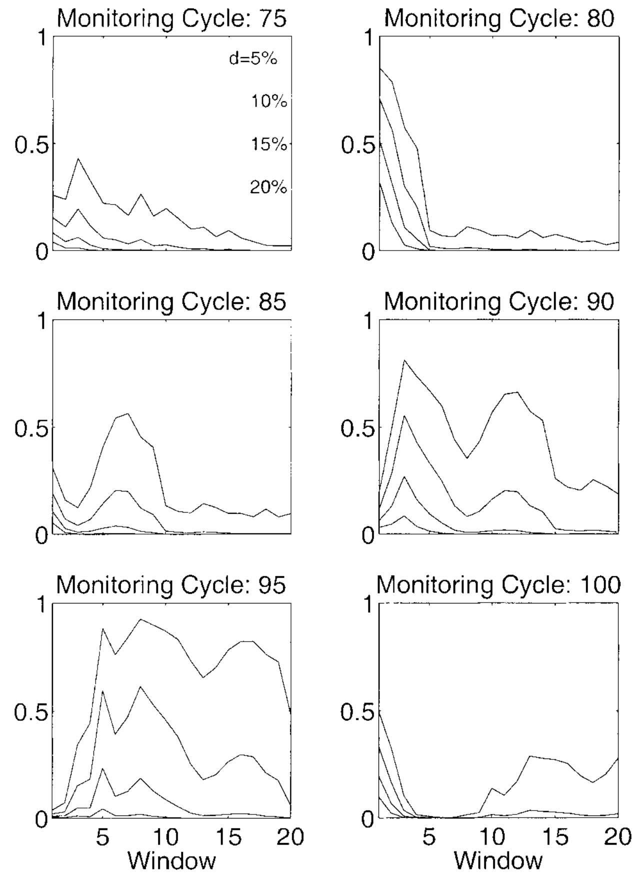

FIG. 6. Monitoring History for P_4^{dam} for Larger Number of Monitoring Cycles

considering the damage measure, and in general damage detection results, using different amounts of available information (rather than just the most recent data), so that the effects of noise are mitigated. The damage measures in Fig. 6 are generally small for different monitoring cycles (except monitoring cycle 95), indicating no substantial damage in the fourth story. Note that the 95th cycle, the damage measure is quite high for d_4 = 5%, which seems to indicate a 5% damage. However, this observation does not persist in subsequent cycles.

The manner in which the alarm function is exceeded provides some way to distinguish levels of damage. For large levels of damage in the *i*th substructure, the P_i^{dam} will quickly be driven to 1 for all *k*, so such damage is quickly detected. For moderate levels of damage, P_i^{dam} will not shift to 1 immediately, but will still tend toward 1 for most of the *k*. Low levels of damage will not cause P_i^{dam} to exceed the alarm level for small values of *k* with few monitoring cycles. However, as more damaged data is acquired, the probability of variation should begin to rise above the alarm level for large values of *k*, since the effects of noise are being reduced. Therefore, small levels of damage may be eventually detected by monitoring the structure over longer times and tracking the behavior of the P_i^{dam}. Alarms when there is no damage do not show the behaviors described for the damaged cases. Thus, recognition of these features can be programmed into an expert system to assist in the verification of an alarm when one is set.

Note that considering the data over long periods of time will not mitigate regularly persistent variations such as those due to diurnal changes. Suppose, however,that such effects can be observed while the structure is in its undamaged state, and different sets of ''undamaged'' data can be associated with different environmental conditions. Then, different undamaged PDFs formed from these data sets can be used as the reference PDF depending on the conditions during the modal tests. This is one way in which this type of model error can be accounted for in the proposed SHM framework.

The few examples depicted here illustrate the behavior and the use of the probabilistic SHM method. Other cases were run with only partial modeshape information and more modes (Vanik 1997). Damage could still be detected in most cases. In a few cases with limited modal information, however, potential damage was indicated in undamaged substructures.

CONCLUSIONS AND FUTURE WORK

This paper has discussed the issues associated with uncertainty in applying SHM to real structures and presented a probabilistic method for continual on−line implementation that takes these factors into consideration. A novel approach to monitoring which involves studying the variation in time of a probabilistic damage measure was introduced. This approach may enable small levels of damage to be detected through monitoring of the structure over long times. Some preliminary results of testing on simulated data were shown, but more thorough testing of the strengths and limitations of the proposed probabilistic SHM approach is required. The method must be tested on more complex simulated structures, and then with data from real structures. Also, the variation of P_i^{dam} with and without damage should be further characterized so that a set of rules for use by an expert system or end user can be established. Finally, if this additional work is successful, software for implementation of the SHM procedure in an automated fashion must be developed to provide real−time monitoring.

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